How were Western written calculations introduced into China? — An analysis of the *Tongwen suanzhi* (*Arithmetic Guidance in the Common Language*, 1613)

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The *Tongwen suanzhi* (*Arithmetic Guidance in the Common Language*, 1613) has always been viewed as the first mathematical book to introduce Western written calculations into China. By analyzing the way in which calculations were written in the *Tongwen suanzhi*, this paper reveals that a place-value number system using Chinese characters was used to record operands and results in the calculations; however, intermediate calculations such as additions and subtractions were not recorded in the text. Therefore, when people in the 17th century read this book, they probably used an abacus as an aid to perform intermediate calculations for the sake of convenience. As a result, on the one hand, Western written calculations were changed in the process of transmission, and on the other hand, the writing mode in the *Tongwen suanzhi* influenced the way in which Chinese people studied mathematics in the Qing dynasty.

**KEYWORDS**
abacus, Chinese characters, Li Zhizao, Matteo Ricci, Tongwen suanzhi, Western written calculations

1  |  INTRODUCTION

The 17th century played a key role in the history of cultural exchange between China and the West. With the arrival of the Jesuits, Western knowledge concerning various disciplines was introduced into China. For example, in mathematics the translation of the first six books (juan 卷) of Euclid’s *Elements* (its Chinese title: *Jihe yuanben* 几何原本) by Matteo Ricci (1552–1610) and Xu Guangqi 徐光启 (1562–1633) in 1607 was the...
most significant work of its kind and introduced Euclidean geometry to China. Moreover, the Tongwen suanzhi (同文算指 Arithmetic Guidance in the Common Language), translated and compiled by Matteo Ricci and Li Zhi-zao 李之藻 (1565–1630), was the first treatise introducing Western arithmetic to China, in particular Western written calculations.2

It was usually believed that the composition of the Tongwen suanzhi was mainly based on Christopher Clavius’s (1538–1612) Epitome Arithmeticae Practicae (1583).3 Recent research, however, reveals that the sources of the book are various, including other Western texts such as Michael Stifel’s (1487–1567) Arithmetica Integra and some Chinese mathematical texts, such as Cheng Dawei’s 程大位 Suanfa tongzong (General source of mathematical methods, 1592) and Zhou Shuxue’s 周述學 Shendao dabian lizong suanhui (General compilation of God’s universal principle of calendric sources and mathematical collections, 1558).4 In fact, Ricci and Li began translating Epitome Arithmeticae Practicae as early as 1603 and completed the work in 1608. After the Jesuit’s death in 1610, Li continued discussing the treatise with Xu Guangqi.5 The first printed edition of this book was published in 1613. Therefore, it is reasonable to consider the book’s sources as coming from the West and China.

In the scholarly community, the Tongwen suanzhi is often viewed as the first book to bring Western written calculations into China.6 Although this statement is generally true, the basis of the original text has not received a detailed and careful analysis. On one hand, Epitome Arithmeticae Practicae was in Clavius’ mathematical curriculum. Its translation was part of the plan to build the entire Jesuit educational system in China.7 On the other hand, when the authors were composing the Tongwen suanzhi, the abacus had already replaced counting rods as the main computational instrument in Chinese mathematics.8 Therefore, the history of using an abacus in China should be seriously considered. An ancient Chinese mathematical book, titled Shushu jiyi (數術記遺 Notes on Traditions of Mathematical Methods), written by Xu Yue 徐岳 in the late second century or early third century, recorded the “ball-arithmetic” (zhu suan 珠算). However, the shape and the structure of this kind of abacus are different from the well-known Chinese abacus.9 The abacus entered into common use in China only during the second half of the 16th century. It is very probable that this abacus was derived from counting rods.10 All these facts reflect the complexity of mathematical instrument change in the early 17th century.

This article aims to carefully analyze the writing of calculations in the context of using Chinese characters as in the Tongwen suanzhi. Also analyzed is the way the abacus and written calculations fit together in this process. My aim is to reveal how Western written calculations were replaced, modified, or hybridized when they were introduced into China. I will argue that the missing information in the calculation process is mainly intermediate calculations such as addition and subtraction; for 17th century readers, these calculations could be performed with the abacus for the sake of convenience.

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2Counting rods (suan chou 算筹) were the main instruments used in Chinese mathematics from ancient times to at least the 15th century. Li Di, 1991, 2002 argued that written calculations already appeared in a thirteenth century mathematical treatise, Shushu jiuzhang (Mathematical Book in Nine Chapters 数書九章, completed by Qin Jiushao in 1247). However, the written numerals were derived from counting rods. Zheng Cheng & Zhu Yiwen, 2010 further argued a linear system existed in Shushu jiuzhang to represent different operations. Li Di and Feng Lisheng also argued that two systems were used for written calculations in the Qing dynasty (1644–1911): one was the new rod-signs system derived from the 13th-century mathematical writings, and the other was based on the Western system introduced in the 17th century.

3For example, Guo Shuchun, 2010, pp. 616–618.


6The statements appeared in almost all the secondary sources mentioned above.

7Meynard, 2017.


In most editions, the *Tongwen suanzhi* consists of the following three parts: "preceding compilation" (qian bian 前編), "general compilation" (tong bian 通編), and "additional compilation" (bie bian 別編). The "preceding compilation" has 2 books. Book 1 is further divided into five parts: "determining positions" (ding wei 定位), "addition methods" (jia fa 加法), "subtraction methods" (jian fa 減法), "multiplication methods" (cheng fa 乘法), and "division methods" (chu fa 除法). Book 2 addresses operations involving decimal numbers and fractions. The "general compilation" has 8 books including the rule of three, the rule of the false double position, linear equations, square and cubic root extraction, etc. The "additional compilation," which has no printed edition, has only 1 book and relates to some astronomical problems. In this study, I will focus on book 1 of the "preceding compilation," which not only directly relates to written calculations but also serves as a basis for the entire book.

### 2.1 A new written number system using Chinese characters

The main body of the *Tongwen suanzhi* begins with the "determining positions." It presents the number system. This section starts with an introduction to different instruments: counting rods, an abacus, and written calculations. It begins as follows:

古法用竹，徑一分，長六寸，二百七十一而為六尺，為一握。度長短者，不失毫釐；量多少者，不失圭撮；權輕重者，不失黍絫。紀於一，協於十，長於百，大於千，衍於萬，算之原也...

The ancient method (of making counting rods) uses bamboo. The diameter (of the cross-section of counting rods) is one fen 分, its length being six cun 寸. Two hundred and seventy-one counting rods form the shape of a hexagon that is a handful. When one uses them to measure length, it will not lose in hao 毫 and li 米 (two small units for length); when one uses them to measure capacity, it will not lose in gui 圭 or cuo 撮 (two small units for capacity); when one uses them to measure weight, it will not lose in shu 筒 or lei 磊 (two small units for weight). Recording a unit (yi 一, a character to indicate unit place), harmonizing tens (shi 十, a character to indicate tenth place), extending one hundreds (bai 百, a character to indicate hundredth place), greater one thousands (qian 千, a character to indicate thousandth place), and inferring ten thousands (wan 萬, a character to indicate ten thousandth place), that is the origin of mathematics...

The counting rods described in this paragraph have a circular cross-section whose diameter is 1 fen (=0.231 cm) and have a length of 6 cun (=13.86 cm). Li Yan interpreted how 271 counting rods form the shape of a hexagon, as shown in Figure 1. The sentences from “the ancient method” to “the origin of mathematics” are composed on the basis of the “Lù-lǐ zhī” 律曆志 (Monograph on Harmonics and Calendars) of the *Book of Han*, which discusses the system of counting rods in the Han dynasty. The main difference between the *Tongwen suanzhi* and the *Book of Han* is that the latter reads, “their methods lie in mathematical procedures” while the former says, “that is the origin of mathematics.” This difference reveals that the authors of the *Tongwen suanzhi* made use of early records on counting rods to discuss the origin of mathematics. In fact, a similar part discussing a different system of counting rods in later...
times was presented in the “Lü-li zhi” of the Book of Sui, which was written by Li Chunfeng 李淳風 (602–670 CE). However, the authors of the Tongwen suanzhi did not mention Li Chunfeng’s records.17

After discussing counting rods, the authors say that the abacus was made for convenience. However, position determinations remain very difficult. Hence, people use writing. It reads as follows:

...後世乃為珠算，而其法較便。然率以定位為難，差毫釐，失千里矣。茲以書代珠，始於一，究於九，隨其所得而書識之。滿十則不書十，而書一，於左進位，乃作于本位。一〇曰一十。由十進百，由百進千，由千進萬，皆如此。

...Later, generations use the abacus, which is more convenient. However, in mathematics (lù 率) the determination of positions is difficult. When deviating in hào 毫 and lǐ 里, the result will lose one thousand lǐ1里.18 Therefore, one uses writing to replace the abacus. The written system starts from one and ends with nine. With what is obtained, one writes it for recognition. When completing one ten, one does not write ten; one writes one on the left high position and writes 于 in the original position. One (after which a) 于 means ten. From tens one advances to hundreds, from hundreds one advances to thousands, and from thousands one advances to ten thousands all in imitation of this.19

The advantages and disadvantages of the abacus are stated in this paragraph. In the authors’ view, the advantages of the abacus explain why the abacus replaced counting rods; whereas its disadvantages in determining positions causes the use of writing. In what follows, the authors state the new written number system using Chinese characters.

As is known, numbers written with Chinese characters commonly do not use a place-value system. For instance, to represent 25, one cannot only write two characters: two (二) five (五). Instead, one should write three characters: 2 (er 二) 10 (shi 十) 5 (wu 五) to represent 25. Shi gives the previous “two” the meaning of 20. Such characters as 1 (yi 一), 10 (shi 十), 100 (bai 百), 1,000 (qian 千), and 10,000 (wan 萬) are all characters for determining positions. This information is exactly what was recorded in the Book of Han and quoted by the authors of the Tongwen suanzhi. However, in the following, the authors explain how to make the Chinese characters a place-value system. In this sense, the authors

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17One conspicuous feature of Li Chunfeng’s records is that he reports two kinds of counting rods, positive and negative, which was not stated in the Book of Han. See Book of Sui, 16.387. The fact that the authors of Tongwen suanzhi did not use Li’s record may also reflect that the two kinds of counting rods are not important in this book.

18According to the regulations in the Qing dynasty, 1 lǐ 里 = 180 0000 lǐ 里 = 1800 0000 hào 毫.

19Tongwen suanzhi, 1.80.
TABLE 1  A new written number system using Chinese characters

<table>
<thead>
<tr>
<th>Hindu-Arabic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese Characters</td>
<td>一</td>
<td>二</td>
<td>三</td>
<td>四</td>
<td>五</td>
<td>六</td>
<td>七</td>
<td>八</td>
<td>九</td>
<td>0</td>
</tr>
</tbody>
</table>

change the use of common Chinese number characters. In fact, they only use nine characters: one (yi 一), two (er 二), three (san 三), four (si 四), five (wu 五), six (liu 六), seven (qi 七), eight (ba 八), and nine (jiu 九) and add a circular symbol, 〇, which means zero. The new system can be compared with the Hindu-Arabic numbers, as shown in Table 1.

The authors further give an example of how this new written number system can be used. The number, 43,210, is written using numbers with the “place-value” Chinese characters, as shown in Figures 2 and 3. Below each character are two small characters to explain the number positions above. However, we shall see that these two characters below are not written when doing calculations. Therefore, they are only used for explaining single numbers.

In a 13th-century mathematical book, titled Chengchu tongbian benmo (乘除通變本末 Records on the General Change of Multiplication and Division, 1274), the author Yang Hui 楊輝 used the following three number systems: first is the place-value rod signs, derived from counting rods; second is the numbers only written in common Chinese characters, which is not a place-value system; and third is the numbers written in unfalsifiable Chinese characters, such as 壹, 貳, 參, etc., which is a place-value system. Hence, the number system in the Tongwen suanzhi can be viewed as a continuation of Yang Hui’s writing, but such a use of common Chinese characters is really new.

Below Figure 2, the authors explain in detail the writing direction and measuring units used for different positions. Numbers in this system are in fact written from left to right. Moreover, on the left, positional characters are written to indicate increasingly higher positions, while on the right characters for weight units are recorded to indicate increasingly smaller positions. The explanation reads as follows:

自左方寫起,平行,大數列左,小數列右。若從小數起積者,每滿十則進位,一十者書一,二十者書二,餘仿此。若大數積多,則於左方漸進加字,如後圖,萬、億、兆、京是也。若小數積多,則於右方漸退加字,如兩下有錢,錢下有分,分下有釐,又有毫、有絲、有忽之類也。

Start writing from the left, moving horizontally. Numbers in higher positions (da shu 大數) are on the left, and numbers in lower positions (xiao shu 小數) are on the right. If while accumulating numbers in lower positions, every time (a position is) filled up to a ten, (the ten) advances to the higher position. One ten being written as one, two tens being written as two, and so on. If numbers in higher positions accumulate, then gradually add characters on the left side, as in the following figure, which is ten thousand (wan 萬, a character to indicate ten thousandth position), one hundred million (yi 億, a character to indicate hundred millionth position), one trillion (zhao 兆, a character to indicate trillionth position), and one hundred million trillion (jing 京, a character to indicate hundred million trillionth position). If many numbers in lower positions are accumulated, then gradually add characters down the right, for example below liang 錢 being qian 錢, below qian being fen 分, below fen being li 釐. Additionally, there are hao 蒙, si 絲, and hu 忽 (liang, qian, fen, li, hao, si, hu are all measuring units for weight).

At the end of the section on "determining positions," the authors give another example of a very big number with 25 digits, 4629541869735243806195634. As previously, the authors explain in detail the different measuring units used for different positions below this larger number. These measuring units are for length, area, capacity, weight, and even astronomical computation. The authors further compare the different units in different times. The authors

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20Chenchu tongbian benmo, 1049–1072.
21Although the new number system reads from left-to-right instead of from right-to-left, the writing order follows the system of counting rods and Yang Hui’s system. Later, Mei Wending (1633–1721) changed the writing direction from horizontal to vertical to fit with typical Chinese writing.
22Tongwen suanzhi, 1.80.
give this example because they want to show a more general use of the new written number system. After this example, the preparation of the written number system in the Tongwen suanzhi is completed.

2.2 | Written addition using Chinese characters

Based on the new written number system, the Tongwen suanzhi continues to explain four operations—addition (jia 加), subtraction (jian 减), multiplication (cheng 乘), and division (chu 除). Since the detailed use of characters in calculations has not been fully studied, this will be the focus of this article. Among the four operations, addition is considered first, followed by the other three operations.²³

Following the structure of the section on "determining positions," the section on "addition methods" also introduces a general rule of addition; then, it gives some examples. In the beginning, the authors say:

凡數惟加法最易。加之不已，至於無算。故算首論加。加也，併也，積也，一也。少曰併，多曰積，皆加也...

Among all operations, the easiest one is addition. Addition is done continuously until no calculation (needs be done). Therefore, addition is first discussed in mathematics. To add (jia 加) is to combine (bing 併), to accumulate (ji 積), or to unite (yi 一). When numbers (to be added) are less, the operation is called bing; when numbers are more, the operation is called ji. Both are addition...²⁴

First, the authors state the feature of addition, and give four different names (jia, bing, ji, yi) to the different kinds of additions. Jia and yi both are general names for addition, although the latter is seldom used in texts. The difference

²³One could understand that addition is the most fundamental among the four operations, since principally subtraction could be seen as the inverse of addition, multiplication as the repetition of addition, and division as the inverse of multiplication; moreover, the key sub-procedures for multiplication and division are addition and subtraction.

²⁴Tongwen suanzhi, 1.81.
between bing and ji lies in whether the operands are few or many. In what follows, the authors give two examples: first, four numbers to be added are called bing; second, twelve numbers to be added are called ji. This difference shows the number of operands is also a key factor for addition. This factor will influence the convenience of the operation, and may be a hint to analyze the calculations conducted outside the text. After this, the authors continue writing:

...列散數於上,各橫位以類相等如十從十,百從百,及兩從兩,斗從斗之類先從小數併之，而以所得數紀於本位。下遇十則進一位,遇百則進二位。

...One places the scattered numbers (on the paper/board), and horizontally places them according to their category. [Commentary: For example, that in the tenth position adds to the other in the tenth position, that in the hundredth position adds to the other in the hundredth position, and that in the liang's position adds to the other in the liang's position (liang 两 is a measuring unit for weight), and that in the dou's position adds to the other in the dou's position (dou 斗 is a measuring unit for capacity), etc.] One first adds numbers in lower positions and writes what was obtained in the original position. When one meets with ten below, one moves up one position; when one meets with one hundred below, one moves up two positions.25

The operands in addition are called scattered numbers (san shu 散數). The general method given for addition has four steps: (a) to make the operation in the proper positions; (b) to start adding from the small numbers; (c) to move up to higher positions when having 10, 100, etc.; and (d) to record the result in the original position. In what follows, two examples with diagrams are given to illuminate the general rule. I will analyze them individually.

As motioned above, the first example is to do the bing addition: $7,10,654 + 8,907 + 56,789 + 880 = 77,7230$. Numbers are written in the place-value Chinese characters. The authors write the sentences below the added numbers to explain every step of the addition as seen in Figures 4 and 5:

The addends are written above, and the sum is below. A line is used to divide the two parts. The authors say after the addition, “The right scattered numbers are four items, which are placed on the above grid. Adding them makes the sum seven hundred seventy and seven thousand two hundred and thirty, which numbers are placed below.”26 This shows the paper for addition is divided into grids. The grid above is used to write addends, and the one below is for the sum. However, although the explanation is detailed, the authors do not give the process for the intermediate calculations, that is, one-digit addition. This omission leaves us with the question of how to do these kinds of basic calculations. When we realize that in the seventeenth century the abacus had replaced counting rods as the dominant mathematical instrument, three possibilities remain: written calculations, the abacus, and mental calculations. I will address this issue below.

Let us turn to the second example, which is a ji addition with 12 numbers, as shown in Figures 6 and 7: $6,008 + 5,009 + 4,009 + 308 + 239 + 108 + 108 + 309 + 4,128 + 3,009 + 209 + 308 = 23,752$. Additionally, a line is used between the paper’s two parts.

Because the addends are many, the authors explain only the result of each addition step. Moreover, mental calculations are neither easy nor convenient for this kind of addition, for instance, the addition of the units $8 + 9 + 9 + 8 + 8 + 8 + 9 + 8 + 9 + 9 + 8 = 102$. Hence, the method for performing this intermediate calculation remains unanswered.

After the two diagrams, the authors say, "Two figures above exhaust addition."27 This shows the authors believe the cases in the two diagrams have covered all cases of addition. In what follows, two methods for checking the answers are

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25 Tongwen suanzhi, 1.81. Sentences in square brackets are the authors’ comments, which were written in small characters in the Chinese text.
26 右式散數四項，列格上。併總得數，七十七萬七千二百三十，列格下。Tongwen suanzhi, 1.81.
27 以上二圖盡加法矣。Tongwen suanzhi, 1.82.
presented. One checking method relies on the additive commutative law, that is, $a + b = b + a$, and then compares the two sums. The other checking method is to use the inverse operation of addition, that is, subtraction. One adds what has been subtracted, that is, $a + b - x + x = a + b$. The general principle of the checking methods (shi fa) reads as follows:

**One method:** one calculates from numbers above to below, obtaining some quantity. Further, one calculates from numbers below to above, obtaining some quantity. One records the sums (to compare).

**Another method:** one checks addition by subtraction, causally subtracting one row, obtaining some quantity, and then adding what was subtracted resulting in the same quantity.

In addition to these general methods, the authors also give two specific methods, which are to use two numbers—7 and 9—and to subtract. The idea is that all the addends divided by 9 or 7 are congruent to their sums divided by 9 or 7: if $a + b + c = s$, then $a + b + c \equiv s \pmod{9 \text{ or } 7}$. It reads as follows:

<table>
<thead>
<tr>
<th>FIGURE 4  (source: Tongwen suanzhi, 1.81)</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image](source: Tongwen suanzhi, 1.81)</td>
</tr>
</tbody>
</table>

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28 另有試法具後. The authors say: “Checking methods are stated below.” Tongwen suanzhi, 1.82.

29 Tongwen suanzhi, 1.82.
Further, when dealing with the difference between scattered numbers and the sum, there are two methods: subtraction by nine or subtraction by seven. One first subtracts the scattered numbers, with some quantity remaining. One second, subtracts the sum, with some quantity remaining. One lists and compares the two remaining numbers. If they are the same, the calculation has no error. If they are different, the calculation has errors.

Since the methods are not easy to understand, the authors give four examples below, based on the previous two additions shown in Figures 4–7 for showing two checking methods on each one. Figures 8 and 9 are the examples of the checking method, using 9 to subtract. However, looking at the diagram, the sentence below gives only the general principle of the checking method. If one reads only this figure and the text, the method is impossible to understand. Relying on the other three examples, one can understand the details of the method, which can be understood in modern terms: the aim of the method is to check whether $710,654 + 8,907 + 56,789 + 880 = 777,230$ is correct. On the left side of the equation, we have $710,645 \equiv 5 \pmod{9}$, $8,907 \equiv 6 \pmod{9}$, $56,789 \equiv 8 \pmod{9}$, $880 \equiv 7 \pmod{9}$, so $5 + 6 + 8 + 7 = 26 \equiv 8 \pmod{9}$, and on the right side of the equation, we have $777,230 \equiv 8 \pmod{9}$. Hence, the calculation is correct. This demonstration explains why two eights rest on both sides of the cross. In fact, Clavius explains in detail how the checking method works in the corresponding part in the *Epitome Arithmeticae Practicae*.

This fact not only raises the question of how Matteo Ricci and Li Zhizao made the translation but also raises the question as to how the checking method in the *Tongwen suanzhi* was conducted.

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2.3 | Features of writing and other operations

Based on the previous analysis, we can summarize the structure of the section on "addition methods," which can be viewed as a model for other sections in the Tongwen suanzhi, as follows:

1. A general discussion on the role of addition, involving the technical terms related to addition, and a general description of the addition method.
2. Based on the addition method, two specific examples of addition are given. These examples are written using diagrams, with the authors' commentaries written below. However, the intermediate calculations are not presented in the text.
3. A general discussion on the principle of the checking methods is presented. Two specific checking methods are given.
4. Based on the examples of addition and the checking methods, four examples using checking methods are given. These examples also contain diagrams and the authors' commentaries. However, also no presentation is given for the intermediate calculations in the text.

This structure reveals two features of the Tongwen suanzhi's writing style. The most obvious one is the combined use of diagrams and commentaries. This feature also exists in the mathematical documents of the Song and Yuan periods (960–1368) and is necessary when the written calculation must be introduced. Only by using diagrams can we understand the method directly, and only by relying on commentaries can we understand the diagrams fully.
Another feature of the writing style is to present the general methods at first and then some specific examples based on these methods. This method is a traditional way of writing a mathematical treatise and was used in, for example, The Nine Chapters on Mathematical Procedures (九章算術, hereafter The Nine Chapters), the most important mathematical treatise from ancient China. This feature reveals abstraction and generality as epistemological values for mathematical methods. In the "subtraction," "multiplication," and "division" methods, the authors follow the same writing style. For subtraction, the authors say at the beginning:

減與加反,用稽所餘。其法先較數之多寡,多中減寡,亦自右方小數減起,以漸進位。其辨多寡之法,於左方首位辨之,首位相等,乃視次位,次復相等,逐位進求,則多寡分焉。

Subtraction, which is the reverse of addition, is used to investigate the remainder. The method is to compare the amount of (two) numbers. The smaller number is subtracted from the larger number, and subtraction begins from the lowest position, on the right, and gradually advances through [higher and higher] positions. The method for comparing the amount of (two) numbers is to compare from the first positions in the left; if the first positions are equal, one checks the second positions; if the second positions are equal, one further looks down at the other positions one by one. Thereby, one differentiates the amount of (two) numbers...

31 Tongwen suanzhi, 1.83.
In this paragraph, the role of subtraction is first stated. Second, a method for comparing two numbers is given. The aim of the method is to differentiate the minuend and the subtrahend, respectively called 原数 and 减数. In the first chapter of The Nine Chapters, a procedure—namely, comparison of fractions (课分)—is used to compare two fractions. The procedure’s key is to make two fractions’ denominators equal and compare their numerators. However, The Nine Chapters does not offer a procedure for this, which can be done exactly using the authors' method here. Therefore, the comparison method the authors give is more basic. After this, the authors also use diagrams to give examples of this procedure. However, the intermediate calculations, which are additions and subtractions, are still not written. After the diagrams, the authors give two checking methods for subtraction, whose principle is similar to the addition cases.

In the passages on multiplication and division, presented successively are the general methods, the detailed examples using diagrams and commentaries, and the checking methods for the two operations. For multiplication, the authors first state multiplication's role. They further clarify two technical terms for multiplication (因 and 乘), which is an ancient difference. The opening paragraph also indicates the key to multiplication, which is the Nine-Nine table. Similarly, to division, two technical names of division (除 and 防) are first clarified, followed by the method

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33 Before the diagrams are presented, the text reads: "Since one examines the amount of (two) numbers, the original number (that is the minuend, 原数) is placed above, and the subtrahend (减数) is placed below. Using the method from the right, the remainder is recorded below one by one. If a smaller number is subtracted from a bigger number, one need not use the borrowing method, just like the first diagram in the following. If a larger number is subtracted from a smaller number, one has to set up the borrowing method, and communicates its change, just like the second diagram in the following" (Tongwen suanzhi, 1.83).
34 The first paragraph on multiplication reads, "Already knowing addition and subtraction, it is time to discuss multiplication (因 and 乘). When (the multiplier is) one-digit, it is called yin. When (the multiplier is) multi-digit, it is called cheng. Generally, they are called cheng. The subtlety of multiplication lies in the Nine-Nine (that is the multiplication table). Make the diagram of the Nine-Nine..." (Tongwen suanzhi, 1.85).
of placing dividends and divisors. The authors also refer to the order of operations and the method of moving back one position.35 Some scholars suggest the multiplication method in the Tongwen suanzhi was derived from the ancient multiplication system using counting rods. However, the division method was a galley method, which was popular in the West at that time.36 This method was first introduced into China in the Tongwen suanzhi.37 Generally, these statements are acceptable. Nevertheless, whether in multiplication or division, the intermediate calculations are not written. Western methods for the four operations are written calculations, relying only on the use of pen and paper. Almost all required information, including intermediate calculations, is written.38 However, this procedure is not the case in the Tongwen suanzhi as shown in Figures 4–9, since the information given is incomplete. In general, for all four

35The first paragraph on division reads, "For all procedures in which small numbers are used to cut big numbers, it is called chu 除 (that is to divide), and it is also named gui 归 (that is to classify). Chu and gui are both to divide what has been obtained. Chu is to split into parts (fen 分), to divide (chu 除), or subtract (jian 减), their meaning being one and the same. The method is to place the dividend (yuan shu 原数) on the upper layer, place the divisor (chu shu 除数) on the second layer. The old method takes the yuan shu as the dividend and the chu shu as the divisor. Dividing from the numbers in higher positions (da shu 大数), two numbers above and below are placed closely. However, the numbers in lower positions (xiao shu 小数) must be attached below the numbers in higher positions. If the dividend on the above layer is small, and the divisor on the below layer is large, one should move back one position, which has been explained in detail on the left..." 凡数以少剖多,曰除,亦名归。除归者,各分所入。除者, 分, 减, 其义一也。法列所数於上层,列除数於次层。舊以原數為實, 除數為法。從左大數列起, 下下換身列位。然必以小數系大數下。若上層原數小, 下層除數大者, 須退一位, 系之詳具左 (Tongwen suanzhi, 1.90).

36Before 1600, the most common method of division used in Europe was the galley (or batello or scratch) method. The method was called "galley" since the outline of the work was thought to resemble a boat. See Lam Lay-Yong, 1966. I think the differences between multiplication and division in the Tongwen suanzhi could be attributed to the special position of division in Chinese mathematics. See Chemla & Guo Shuchun, 2004, p. 911.

37Lam Lay-Yong, 1966; Man Keung Siu, 2015.

operations in the Tongwen suanzhi, the authors did not write the intermediate calculations that are addition and subtraction. Previous studies raise a hypothesis that the methods were fully transmitted into China in the beginning of the 17th century.\textsuperscript{39} This point needs to be re-thought now.

Three possibilities exist for the calculations conducted outside the text—written, the abacus, or mental—and the answer could be different for different people. For the authors of the Tongwen suanzhi, since their aim is to introduce Western method, they may have also used written calculations for the intermediate calculations. However, for common 17th century readers, these intermediate calculations were probably done with the abacus because of its convenience in adding and subtracting. This point can be confirmed by another source, namely, Taixi suanyao (The Key to Western Mathematics). This book was written by Sun Yuanhua 孫元化 (1581–1632) in the 1620s and was a book for Western written calculations.\textsuperscript{40} The author was a student of Xu Guangqi and was influenced by the Tongwen suanzhi. For convenience, the author states that, “For addition, the abacus is more convenient than written calculations. For subtraction, their conveniences are equal. For multiplication, the abacus is less convenient than written calculations. For division, written calculations are ten times more convenient.”\textsuperscript{41} This statement reveals that the abacus could still be used for addition and subtraction at that time. In fact, almost no intermediate calculations such as additions and subtractions were recorded in the Tongwen suanzhi. Therefore, these missing calculations could be conducted with an abacus if 17th century readers preferred convenience. In other words, after reading the Tongwen suanzhi, readers could use paper and an abacus simultaneously to do calculations.\textsuperscript{42}

3 | HISTORICAL CONTEXT

Since the Tongwen suanzhi is a treatise combining traditional Chinese and Western mathematics, the process of writing and compiling the book should be reviewed, especially the interplay between different cultural elements. To our knowledge, Matteo Ricci and Li Zhizao met each other in 1603. At that time, Ricci was already a well-known Jesuit and had lived in Beijing, the Ming capital, for three years; Li Zhizao was serving the Ming court in Fujian, in the south of China. Soon, Li acquired Western scientific knowledge—e.g., astronomy, mathematics, and geography—from Ricci. Moreover, as an examiner at the provincial level (xiangshi 郷試) in Fujian, Li inserted an astronomical question into the examination that was based on Western learning. This fact strengthened his confidence in Western knowledge.\textsuperscript{43} Li finally became a Catholic in 1610.\textsuperscript{44} From 1603, Ricci and Li are believed to have prepared the translation of the Epitome Arithmeticae Practicae and the compilation of the Tongwen suanzhi. As the Tongwen suanzhi indicates, their cooperative mode was for Ricci to speak (shou 授) and Li to write (yan 演). This process was the same as the cooperative work of Ricci and Xu Guangqi on the translation and compilation of Euclid’s Elements. The translation work of Ricci and Li was completed no later than 1608. However, the book was not published immediately. After Ricci’s death in 1610, Li continued discussions with Xu. Finally, the Tongwen suanzhi was published in 1613 and included more Chinese sources. The book was viewed as the combination of Chinese and Western mathematics.

When the origin of the treatise was traced from Chinese and Western traditions, modern scholars such as George Sarton (1884–1956) considered Christopher Clavius as the most influential teacher of the Renaissance. Clavius joined the Jesuit order in 1555. From 1565 he taught mathematics in Rome and spent more than 40 years in Collegio Romano. Clavius was interested in astronomy and mathematics and, as an astronomer, his most important achievement was the reformation of the Julian calendar. As an astronomer, he also completed many mathematical works, including the edition of Euclidis Elementorum libri XV, Epitome Arithmeticae Practicae, Algebra, Geometria

\textsuperscript{39} Such as Guo Shuchun, 2010, pp. 616–618; Martzloff, 1997, p. 22.
\textsuperscript{40} Shang Zhicong, 1998.
\textsuperscript{41} 加法珠便於筆，減法之便等，乘法除珠不若筆，分法則筆之便也十倍矣。 Taixi suanyao, 1.2a.
\textsuperscript{42} Of course, mental calculations can be done in some cases, but it is impossible for common readers to use it in all cases.
\textsuperscript{43} Hsu Kuang Tai, 2012; Zheng Cheng, 2009.
\textsuperscript{44} For Li Zhizao’s life and his works, see Fang Hao, 1966.
Practica, etc. Many of his books were taken to China by one of his students, Matteo Ricci. Most important here, Clavius almost established an entire mathematical curriculum, called Ordo servandus in addiscendis disciplinis mathematicis, 1579–1580, for the Society of Jesus. In this sense, the translated works of Euclid’s Elements and Clavius’s Epitome Arithmeticae Practicae by Ricci, Xu, and Li were a part of the Jesuit education in China. This process helps to understand the wider context of the writing of the Tongwen suanzhi.

In contrast to the introduction of Euclid’s geometry, however, arithmetic had its own long history in China. No later than 500 BCE, Chinese used counting rods to compute. In the 13th century, Qin Jiushao 秦九韶 (1208–1261) used both rods and pen and paper for computations. Another scholar, Li Ye 李冶 (1192–1279), wrote the process of forming equations using counting rods. Yang Hui 楊輝 (13th century) used different kinds of Chinese characters to record calculations. The abacus used in the Ming dynasty (1368–1644) probably dates from the 13th century, and its use gradually increased. Wu Jing’s 吳敬 (15th century) Jiuzhang suanfa bilei daquan (九章算法比類大全 Complete Work on the Comparison and Analogy of The Nine Chapters on Mathematical Procedures, 1450) was believed to combine counting rods and an abacus. Additionally, the procedure for multiplication, called xie suan (written calculations), was recorded in this book. This process was the gelosia method, whose Chinese name was “covering the floor with a glamorous carpet” (pu di jin 铺地锦), and which could be traced back to the Islamic world. Before the completion of the Tongwen suanzhi, the Chinese usually used the abacus with the aid of pen and paper to compute. This process is obvious when we consider the Fangcheng procedure (a method for solving systems of linear equations), which requires a set of tabular numbers on the calculating surface and cannot directly be conducted with an abacus. For example, in Cheng Dawei’s Suanfa tongzong (算法統宗 General Source of Mathematical Methods, 1592), he always recorded the beginning setting and intermediate results of the Fangcheng procedure but never gives the details of basic calculations. Mei Wending 梅文鼎 (1633–1721) also confirmed that procedures such as Fangcheng or the rule of the false double position should be conducted with both a pen and an abacus. Therefore, modern scholars, such as Qian Baocong (1892–1974), believed this procedure was done with the aid of pen and paper. This also explains why at the beginning, the authors of the Tongwen suanzhi wrote the Chinese history of mathematical instruments, i.e., counting rods, an abacus, pen, and paper.

In summary, Matteo Ricci’s translation of the Epitome Arithmeticae Practicae had the following two meanings: first, to introduce Western arithmetic and second, as a part of Jesuit education in China. Perhaps the book was planned for use as a textbook. Hence, when Ricci and Li prepared the translation, they may have combined the mathematical knowledge from both China and the West. Additionally, when we compare “addition” in the Epitome Arithmeticae Practicae and Tongwen suanzhi, we discover the main difference lies in the information about the written calculations; the former contained more than the latter. In particular, the checking method lost more information in

45Busard, 1976. For further discussion on Clavius’s contribution on the mathematics curriculum, see Smolarski, 2002.
47Pollet, 2014.
48The origin of the Chinese abacus is still not clear. The abacus recorded in Shushu jiyi is different from the common one. One probable conjecture is that the Chinese abacus was derived from counting rods. Moreover, the pithy formula for computations on an abacus is similar to the one recorded in 13th century mathematical writings. Therefore, the abacus’s use could date back to the 13th century. Also see Guo Shuchun, 2010, pp. 406–408; Martzloff, 1997, pp. 215–216.
49Qian Baocong, 1964, p. 138. There is also a debate whether Wu Jing uses counting rods or the abacus. See Guo Shuchun, 2010, p. 536. However, since evidence of the use of both counting rods and the abacus exist in Jiuzhang suanfa bilei daquan, I agree with Qian Baocong’s point that Wu Jing uses both tools.
50The Gelosia method, also known as the “lattice method,” is a method of multiplication that uses a lattice to multiply two numbers. See Man Keung Siu, 2015.
51For a general discussion on Chinese written calculations, see Li Di & Feng Lisheng, 1998; Li Di, 2002.
52Suanfa tongzong, 11.1359–1364.
53Bisuan, 1.1a.
54Qian Baocong, 1966, p. 92.
the Tongwen suanzhi. All of these differences imply that 17th century readers could probably do the calculations outside the text with the aid of an abacus.

4 | CONCLUSION

With Matteo Ricci’s arrival in Macao in 1582, Western learning was introduced into China. In the early 17th century, with the translation of Euclid’s Elements and Clavius’s Epitome Arithmeticae Practicae, mathematics became one of the first subjects influenced by the West. The Tongwen suanzhi’s completion was not so much the introduction of Western written calculations but the communication between the written traditions of China and the West. As a result, the place-value number system using Chinese characters was established. On the basis of this number system, the development of written calculations with the aid of the abacus changed the roles of mathematical instruments. In other words, in the early and middle Ming dynasty, pen and paper helped the abacus in computation. However, in the Tongwen suanzhi, the abacus became the aid of pen and paper. More importantly, Chinese scholars such as Li Zhi-zao gradually established confidence in Western knowledge. The place-value number system using Chinese characters was adopted by later generations, and the Tongwen suanzhi influenced the way in which Qing scholars studied mathematics. Written calculations with the help of the abacus became the typical way of writing mathematical treatises in the Qing dynasty.56

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PRIMARY SOURCES


56However, if we go further in this direction, we also see the Tongwen suanzhi’s limitations. Since this book was not wholly translated in the Epitome Arithmeticae Practicae, the development of Chinese mathematics was effected through the use of paper calculations.


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